

Comments on Gaugino Mass and Landscape of Vacua

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Abstract

In direct gauge mediation, gaugino masses often vanish at the leading order of supersymmetry breaking. Recently, this phenomenon is understood in connection with the global structure of vacua in O’Raifeartaigh-type models. In this note, we further explore a connection between gaugino masses and the landscape of vacua in more general situations, focusing on a few examples which demonstrate our idea. In particular, we present a calculable model with non-vanishing leading order gaugino masses on the lowest energy vacuum.

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1 Introduction

Gauge mediation [1] is a highly predictive and an attractive way of transmitting supersymmetry breaking of a hidden sector to the supersymmetric standard model. Many gauge mediation models are already known (See [2, 3] for reviews). Among them, direct gauge mediation is the model that the flavor symmetries in a hidden SUSY breaking sector are weakly gauged and identified as the standard model gauge symmetries. In most controllable direct gauge mediation models, the scale of supersymmetry breaking \sqrt{F} (the square-root of the F-term of a SUSY breaking field) is much smaller than the messenger scale M . In such models, the leading contributions to the standard model gaugino masses in an F/M^2 expansion can be calculated by a powerful tool, analytic continuation into superspace [4, 5]. However, it often happens in direct gauge mediation models that the leading contributions to gaugino masses vanish even if R-symmetry is broken, firstly emphasized in [6].¹ Since there are no such cancellations for scalar masses, this implies that the gauginos are much lighter than the scalars.² Then, we cannot obtain order 1 TeV gaugino masses and scalar masses at the same time, which causes the standard model hierarchy problem again. Furthermore, recent experimental data severely constrain such a model with the light gauginos [10]. Thus, it is an interesting challenge to generate sizable gaugino masses in direct gauge mediation. The authors of [11] avoided this issue by exploiting a higher energy metastable state (This possibility was recently emphasized in [12, 13]). However, the reason why an uplifted vacuum generates sizable gaugino masses has remained mysterious.

Recently, Komargodski and Shih (KS) shed light on this curious feature of direct gauge mediation and clarified that the pseudomoduli space cannot be locally stable everywhere in order to generate sizable gaugino masses [14]. It is interesting that anomalously small gaugino masses are closely related to the global structure of vacua. It is worth exploring this connection further in more general situations.

In this paper, we survey the connection not only in direct gauge mediation but also in minimal gauge mediation [15]. We exploit non-canonical Kähler potential of messengers and provide a way to generate the leading order gaugino masses while keeping all messenger directions stable everywhere. Furthermore, we construct a minimal-type gauge mediation model with non-vanishing leading order gaugino masses on the lowest energy vacuum.

The rest of the paper is organized as follows. In section 2, we will review a property

¹Anomalously small gaugino mass problem also occur in another class of gauge mediation models such as semi-direct gauge mediation [7]. In this class, sizable gaugino masses can be generated by introducing a strongly coupled messenger sector [8].

²We can suppress the scalar masses by exploiting a superconformal dynamics [9].

in O’Raifeartaigh-type models with canonical Kähler potential and a connection between the global structure of vacua and the leading order gaugino masses. In section 3, we will show how non-canonical Kähler potential of messenger fields alters the stability of messenger directions at a point of the pseudomoduli space. We also provide an explicit model which has non-zero leading order gaugino masses and a pseudomoduli space that is locally stable everywhere. In section 4, we present a calculable model with an additional $U(1)$ gauge interaction which generates the leading order gaugino masses on the lowest energy vacuum.

2 Review of the KS theorem

First, we will briefly describe the existence of a pseudomoduli space discussed in [14, 16] by exploiting a simple example. Let us consider the following Wess-Zumino model with canonical Kähler potential presented in [17],

$$W = \lambda X(\phi_1 \tilde{\phi}_1 + \phi_2 \tilde{\phi}_2) + m\phi_1 \tilde{\phi}_2 + fX, \quad (2.1)$$

where X is a SUSY breaking field and $\phi_1, \tilde{\phi}_1, \phi_2, \tilde{\phi}_2$ are vector-like pairs of messenger fields charged under the standard model gauge symmetries. We can choose parameters λ, m, f as real without loss of generality. On the metastable SUSY breaking vacuum, $\langle \phi_1 \rangle = \langle \tilde{\phi}_1 \rangle = \langle \phi_2 \rangle = \langle \tilde{\phi}_2 \rangle = 0$ and $\langle X \rangle$ is the pseudomoduli space of vacua. That is, it takes an arbitrary value at the classical level, but takes a definite value by quantum effects. In this model, as discussed in [17], the vev $\langle X \rangle$ takes a non-zero value, so R-symmetry is spontaneously broken. As this example, every O’Raifeartaigh-type model with canonical Kähler potential has a pseudomoduli space on every SUSY breaking vacuum.

Next, we will see a connection between the global structure of vacua and the leading order gaugino masses by dealing with the same example as above. The leading order gaugino masses are calculated by analytic continuation into superspace technique such as,

$$m_{\tilde{g}} \sim f \frac{\partial}{\partial X} \log \det \mathcal{M}_F, \quad (2.2)$$

where \mathcal{M}_F is the fermion mass matrix of messengers. In this model, the determinant of the fermion mass matrix is given by $\det \mathcal{M}_F = \lambda^2 X^2$ and has an X dependence, so gaugino masses are generated at the leading order of the SUSY breaking f ,

$$m_{\tilde{g}} \sim \frac{f}{\langle X \rangle}. \quad (2.3)$$

On the other hand, in this model, there is a zero eigenvalue in the fermion mass matrix at $\langle X \rangle = 0$. Here, the eigenvalues of the scalar mass matrix are $\left(m^2 \pm \sqrt{m^4 + 4\lambda^2 f^2}\right)/2$, so

there is a tachyonic direction. As this example, there is always at least one tachyonic direction on the pseudomoduli space of vacua when gaugino masses are generated at the leading order. Otherwise, the leading order gaugino masses vanish. For general arguments, see the original paper [14].

3 General cases

As we reviewed in the previous section, in every renormalizable O’Raifeartaigh-type model, the pseudomoduli space cannot be stable everywhere to generate gaugino masses. However, a renormalizable model is not always a good description of dynamical SUSY breaking at low-energy. In many SUSY breaking models, correction terms in Kähler potential are not negligible. In this section, we emphasize that such terms affect crucially the connection between gaugino masses and the landscape of vacua.

3.1 Stability of messenger directions

Let us start with a general argument for the stability of messenger directions. Suppose we have a superpotential interaction,

$$W = \mathcal{M}_F(X)_{ab} \tilde{\phi}^a \phi^b + f(X), \quad (3.1)$$

where X is a chiral superfield which is responsible of SUSY breaking and $\phi, \tilde{\phi}$ are messengers. The lower indices of the messenger mass matrix \mathcal{M}_F denote the derivatives with respect to messenger fields. When we turn on a generic non-canonical Kähler potential, X direction is not necessarily pseudo-flat as discussed in [16].³ Nevertheless, in order to focus on the stability of messenger directions at a point of the pseudomoduli space like [14], we can keep a flat direction by imposing the following condition on the metric [18],

$$\partial_X g^{X\bar{X}}|_0 = 0, \quad (3.2)$$

where $|_0$ denotes $\langle \phi^a \rangle = \langle \tilde{\phi}^a \rangle = 0$. It is easy to check that the scalar potential with this condition keeps X direction flat.

In this setup, the boson mass-squared matrix of the messengers is given by

$$\mathcal{M}_B^2 = \begin{pmatrix} (\mathcal{M}_F^* \mathcal{M}_F)_{a\bar{b}} - \mathcal{A}_{a\bar{b}} & \mathcal{F}_{ab}^* \\ \mathcal{F}_{\bar{a}\bar{b}} & (\mathcal{M}_F \mathcal{M}_F^*)_{\bar{a}\bar{b}} - \mathcal{A}_{\bar{a}\bar{b}} \end{pmatrix}. \quad (3.3)$$

Here,

$$\mathcal{F}_{ab} = F_X^* (\partial_X \mathcal{M}_F)_{ab}, \quad \mathcal{A}_{a\bar{b}} = R_{a\bar{b}}^{X\bar{X}} |F_X|^2, \quad (3.4)$$

³Also, D-term SUSY breaking models do not always have pseudomoduli spaces.

where $R_{ab}^{X\bar{X}}$ are components of the Riemann tensor. We simply assumed $g_{X\bar{X}} = 1$ at $\tilde{\phi}^a = \phi^b = 0$. Suppose v_a is a unit vector satisfying $(\mathcal{M}_F)_{ab}v_b = 0$. Then, a bosonic mode corresponding to this direction has a mass,

$$(v^\dagger \ v^T) \mathcal{M}_B^2 \begin{pmatrix} v \\ v^* \end{pmatrix} = v^T \mathcal{F} v - v^\dagger \mathcal{A} v + c.c. \quad (3.5)$$

If $\mathcal{A}v = 0$ or simply if $\mathcal{A} = 0$, then the bosonic mode must be massless in order to have a consistent vacuum, or we have to allow the vacuum to have a tachyonic direction. However, in general, this does not true. As we will demonstrate below, one can easily lift a tachyonic direction and make the pseudomoduli space stable everywhere by using the contribution from the non-canonical part of Kähler potential \mathcal{A} .

3.2 A model with non-canonical Kähler potential

In the rest of this section, we will try to construct the model which has non-zero leading order gaugino masses and a pseudomoduli space that is locally stable everywhere. Let us consider the model discussed in section 2 with non-canonical Kähler potential, that is, its superpotential is given by

$$W = \lambda X(\phi_1 \tilde{\phi}_1 + \phi_2 \tilde{\phi}_2) + m \phi_1 \tilde{\phi}_2 + f X. \quad (3.6)$$

Notation is explained in the previous section. This model with canonical Kähler potential has a tachyonic direction around $\langle X \rangle = 0$, so we will try to lift this direction by introducing non-canonical Kähler potential,⁴

$$K = |X|^2 + \left(1 + \frac{|X|^2}{M^2}\right) (|\phi_1|^2 + |\tilde{\phi}_2|^2) + \left(1 - \frac{|X|^2}{M^2}\right) (|\tilde{\phi}_1|^2 + |\phi_2|^2), \quad (3.7)$$

where M is a large cut-off scale of the theory and we have required vanishing messenger mass supertrace so that our model is UV insensitive [19, 20]. Since the above Kähler potential satisfies the condition (3.2) given in the previous subsection, the pseudo flat-direction of X is kept. There is a zero eigenvalue in the fermion mass matrix at $\langle X \rangle = 0$, and here the eigenvalues of the boson mass-squared matrix of messengers are

$$\frac{1}{2} \left(m^2 \pm \sqrt{m^4 + 4\lambda^2 f^2 - 4(f/M)^2 m^2 + 4(f/M)^4} \right). \quad (3.8)$$

⁴Although we consider a UV-insensitive model and our result does not depend on the way of a UV completion, the authors tried to obtain this kind of non-renormalizable theory from a renormalizable one by integrating out some heavy modes at the tree-level. However, it seems to be difficult to find the UV completion with massive vector bosons of additional gauge groups keeping the flat direction. Integrating out massive chiral superfields also does not seem to be a workable strategy. It might be viable approach to consider the quantum effects of additional fields.

We can impose a condition between parameters of the model such as $\lambda^2 f^2 - (f/M)^2 m^2 + (f/M)^4 < 0$ so as not to have any tachyonic direction. Since non-canonical Kähler potential of messengers does not contribute to the gaugino mass at the leading order, known as gaugino screening [5], the leading order gaugino mass is given in the same fashion as the case with canonical Kähler potential,⁵

$$m_{\tilde{g}} \sim \frac{f}{\langle X \rangle}. \quad (3.9)$$

Here, the expectation value of X can be estimated by stabilizing the one-loop effective potential. The Coleman-Weinberg potential in this kind of models has been calculated in [18], which claims that X does not stabilize at the origin and so R-symmetry is spontaneously broken even in the case with non-canonical Kähler potential. Therefore, we can obtain non-zero leading order gaugino masses in the model with a pseudomoduli space that is locally stable everywhere.

While we have considered a model with a pseudo-flat direction, as we have seen in the previous subsection, the existence of pseudomoduli is not guaranteed in models with non-canonical Kähler potential. In the next section, we will see the relation between this kind of models and the landscape of vacua.

4 Sizable gaugino mass on the global minimum

When we consider the case where there is no pseudomoduli space, it becomes unclear how we can generalize the statement of the KS theorem. We are interested in a connection between the leading order gaugino masses and metastability of the vacuum. Then, we will try to solve the question whether we can obtain non-vanishing gaugino masses on the global minimum or not. The answer is yes. In [21], the authors obtained non-vanishing leading order gaugino masses on the global minimum (See also [22] for a related work on this avenue realizing the ultra-light gravitino mass). However, they used a dynamical SUSY breaking model and the resulting model is in calculable. Then, for our current purpose, we do not need to focus on dynamical SUSY breaking, so we can take our familiar O’Raifeartaigh-type model.

The explicit model of the SUSY breaking sector is a $U(1)$ gauge theory⁶ whose superpotential is given by⁷

$$W = X_0(f + \lambda\varphi_1\varphi_2) + m(X_1\varphi_1 + X_2\varphi_2). \quad (4.1)$$

⁵Unlike the gaugino mass, non-canonical Kähler potential of messengers contributes to the scalar mass at the leading order.

⁶The $U(1)$ gauge coupling becomes strong at high energy. As a logical possibility, there may be a state which has lower energy than the state we now consider by non-perturbative effects. Then, for more rigorous statement, we try to construct a gauge mediation model with the global minimum at least perturbatively.

⁷This model was studied in [23].

The $U(1)$ charge assignments of X_0 , X_1 , X_2 , φ_1 and φ_2 are 0, -1 , 1 , 1 and -1 respectively. We call this $U(1)$ gauge interaction as the messenger gauge interaction. We can take all couplings, λ, m, f as real without loss of generality and assume $f \ll m^2$. On the SUSY breaking vacuum, $\langle X_1 \rangle = \langle X_2 \rangle = \langle \varphi_1 \rangle = \langle \varphi_2 \rangle = 0$ and X_0 has a non-zero F-term.

Next, consider the messenger sector. The simplest possibility for our purpose would be the following,

$$W_{mess} = y_q S q \tilde{q} + y_E S E \tilde{E} + \frac{\kappa}{3} S^3, \quad (4.2)$$

where q and \tilde{q} are messengers charged under the standard model gauge symmetries and S, E, \tilde{E} are the standard model gauge singlets. Only E, \tilde{E} have charges $1, -1$ under the messenger $U(1)$ gauge interaction. We also take couplings y_q, y_E, κ as real. When we integrate out the SUSY breaking sector, two-loop correction generates positive scalar masses for fields E and \tilde{E} like usual gauge mediation, which is given by

$$m_E^2 = m_{\tilde{E}}^2 \sim \left(\frac{g_{mess}^2}{16\pi^2} \right)^2 \left(\frac{\lambda f}{m} \right)^2, \quad (4.3)$$

where g_{mess} is the coupling of the messenger gauge interaction. As pointed out in [21], these positive scalar masses generate negative mass squared by one-loop effects of E and \tilde{E} such as

$$-m_S^2 \simeq \frac{4}{16\pi^2} y_E^2 m_E^2 \ln \frac{\Lambda}{m_E}, \quad (4.4)$$

where Λ is the cut-off scale and we assume $y_E \lesssim 1$ so that $m_E^2 \gg |m_S^2|$ is satisfied. Then, the effective scalar potential of the messenger sector including these corrections is given by

$$\begin{aligned} V_{mess} = & |y_E S \tilde{E}|^2 + |y_E S E|^2 + |y_q S \tilde{q}|^2 + |y_q S q|^2 + |y_E E \tilde{E} + y_q q \tilde{q} + \kappa S^2|^2 \\ & + m_E^2 |E|^2 + m_E^2 |\tilde{E}|^2 + m_S^2 |S|^2. \end{aligned} \quad (4.5)$$

This potential is minimized at

$$\langle |S|^2 \rangle = \frac{|m_S^2|}{2\kappa^2}, \quad \langle q \rangle = \langle \tilde{q} \rangle = \langle E \rangle = \langle \tilde{E} \rangle = 0. \quad (4.6)$$

Note that the expectation value of the SUSY breaking field S is uniquely determined and there is no pseudomoduli space in the messenger sector. The contribution to the vacuum energy is given by

$$V_0 = -\frac{m_S^4}{4\kappa^2}. \quad (4.7)$$

This vacuum is the global minimum in certain parameter range. The standard model gaugino mass can be calculated as

$$m_{\tilde{g}} \sim \frac{\langle |F_S| \rangle}{\langle S \rangle} = \frac{|m_S|}{\sqrt{2}}. \quad (4.8)$$

Therefore, we obtain the leading order gaugino mass on the global minimum of the potential, unlike direct gauge mediation without additional gauge interactions.⁸

While constructing a model with no phenomenological problem is not the purpose of this paper, we finally comment on some points. Although the messenger $U(1)$ gauge boson is massless, it may cause no problem, since the standard model quarks and leptons do not have the messenger $U(1)$ charges. However, if we want to avoid the existence of additional massless gauge bosons, we can higgs the gauge symmetry by adding the following superpotential,

$$W = hT(\Psi\tilde{\Psi} - v^2), \quad (4.9)$$

where $\Psi, \tilde{\Psi}$ are a vector-like pair of chiral superfields charged under the messenger $U(1)$ gauge interaction and T is a Lagrange multiplier field. h is a coupling constant. This term does not restore SUSY, so our argument in this section remains true even if we add this term to the above model. Furthermore, in the SUSY breaking sector, there is a parity given by

$$X_1 \leftrightarrow X_2, \quad \varphi_1 \leftrightarrow \varphi_2, \quad E \leftrightarrow \tilde{E}, \quad V \leftrightarrow -V, \quad (4.10)$$

and so the problematic FI term of the messenger $U(1)$ gauge field is forbidden.

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⁸If we express the gaugino mass in terms of the original SUSY breaking f , the gaugino mass squared comes from a three-loop effect. However, the scalar mass is also suppressed and both masses are almost the same size as in minimal gauge mediation [15].

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